Semi-Simple Sonata Form

An Analysis of Milton Babbitt's 1956, *Semi-Simple Variations*

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Foreward:

This essay revises and expands an analysis I did of Milton Babbitt's *Semi-Simple Variations* for Dr. Amy Bauer's Post-Tonal Music Theory class in March 2009. Although I made a few succinct arguments in my first analysis, I have corrected some mistakes, noticed other significant patterns, produced data tables that demonstrate findings more effectively than prose, and widened the scope of my argument by examining aspects of the piece that are not pitch-related. I have not drawn conclusions from all of the tables I have made, but I have included them for the perusal of others. Throughout my research and analysis, which was conducted in October and November of 2015, I was unaware of Christopher Wintle's 1976 analysis, *Milton Babbitt's Semi-Simple Variations.* Although there are similarities in our analyses, Wintle did not examine the pitch content at three levels, develop tables for pitch content, create an order scheme for the rhythmic partitions, or challenge the structure of the piece.

Introduction:

After extensive research and analysis of Milton Babbitt's *Semi-Simple Variations*, I believe that the piece's form, although presented as a Theme and Variations, is actually written in Sonata form. Numeric reductions of pitch and rhythms clearly demonstrate that the piece consists of two separate, strongly identifiable themes/sections, and that the primary theme returns briefly to close the piece. This essay consists of three sections that present my findings, a conclusion, and appendices of tables that clearly present my argument. Section 1, Pitch-Content Analysis describes how pitch-classes were examined, summarizes the core concepts of

the research, and presents several patterns via the data gathered. Section 2, Rhythmic and Metric Analysis, discusses rhythm and meter in a similar fashion to Section 1. Section 3 *Semi-Simple Variations* as Sonata form, pairs aspects of Babbitt's piece with determining qualities of the traditional Sonata form. The conclusion argues my thesis and presents composition choices made by Babbitt that are not addressed by my thesis.

Section 1- Pitch-Content Analysis:

1.1 Introduction to Pitch: I have analyzed the 12-tone rows and hexachords of *Semi-Simple Variations*; I have not attempted to analyze the pitch content in terms of diatonic Western harmony. In the 1950's, Milton Babbitt was composing highly serial works, and he cites and describes the methods of Schoenberg and Webern as inspiration in his 1955 article, "Some Aspects of Twelve-Tone Composition." Although he may have superimposed instances of diatonic harmony in this complex work, I have not found any strong patterns.

1.2 Three Levels of Pitch Analysis: The pitch content of *Semi-Simple Variations* is constructed of 12-tone rows at three distinct levels that occur simultaneously. Level 1, which produces the clearest results and is wholly constructed of all-combinatorial hexachords, is chorale-style, four-voice part-writing: Soprano, Alto, Tenor, and Bass (at times, this will abbreviated as SATB). Level 2, Upper Voices against Lower Voices, or right hand against left hand, is constructed of three combinatorial hexachords and one semi-combinatorial hexachord. Finally, Level 3 combines all voices (or does not separate notes into voices at all) and simply uses the chronological order at which a pitch-class was sounded to determine its place in the 12-tone row. Level 3 is constructed of many unlike hexachords, some of which do not contain any combinatorial properties.

1.3 Rules for Determining 12-Tone Rows: Babbitt repeated multiple notes throughout this piece, which made recording pitch-class order unclear. To address this complication, I experimented with several different combinations of rules that determined pitch-class order when notes were sounded more than once, held for extended periods of time, or sounded simultaneously with other notes. After many attempts, I arrived at these two:

Rule 1: If new notes are sounded simultaneously, pitch-class order is recorded top to bottom; highest note first, then lower notes in succession. (This rule is not necessary for Level 1 when determining the row of any of the SATB voices, as voices in this style traditionally cannot produce multiple notes at once.)

Rule 2: If a note is repeated at the same register, and it is clear that its Level 1 SATB voice has not moved to another pitch-class, it is not recorded at Level 2 or 3 as a new pitch-class.

1.4 Babbitt's Designated "Theme" vs. Primary Thematic Content: In his program notes, Babbitt declares that [T6E879] is the "Theme" and that its complimentary retrograde, [312504], is the first "Variation". The prime of both of these hexachords is (012345) and, when combined, they create the all-interval, 12-tone row, [T6E879 312504]. Although the pitch content of the two hexachords he cites in his program notes is present in the first three sections, the exact note order of Babbitt's designated "Theme": [T6E879], is only expressed once.

The hexachord (012345) can be considered the primary thematic content because it is present in the expository section, contrasted by a secondary theme and section, and recapitulated to close the piece. It is the prime of all Level 1 hexachords in the Theme and Variation 1, the S and A voices (Level 1) in Variation 2, and the T and B voices (Level 1) in Variation 5. (012345) seems to be an obvious choice for a primary theme because it is a First-Order, all-combinatorial hexachord. Perhaps, (012345) is the simplest and straight-forward combinatorial hexachord, similar to I, or the Tonic, in the diatonic world.

1.5 A Brief Summary of Concepts: There are six all-combinatorial hexachords, and they are the only hexachords capable of reproducing the same collection of six pitch-classes after manipulation by way of Inversion, Retrograde, and Retrograde Inversion at some transpositional level. Semi-combinatorial hexachords reproduce identical pitch-class collections after one or two of these types of manipulation, but not all three. Out of these six all-combinatorial hexachords, three are First-Order, which means they have only one tritone axis with their aggregate hexachord (the six remaining pitch-classes within 12-tone row). The remaining three all-combinatorial hexachords are of different orders, as they have different numbers of tritone axes; Second-Order has two tritone axes, Third-Order has three, and Fourth-Order has six. Combinatorial refers only to pitch content, not pitch order, although several of Babbitt's hexachords in this piece do maintain order after manipulation.

1.6 How to Interpret Appendix 1: The first table is a legend. It pairs repeating prime hexachords with unique colors; all-combinatorial hexachords are in bold face. The three lower tables pair each section with the three levels of interpretation/division of voices explained above. The tables in Appendix 1 only contain reductions of pitch content information; the data presented does not pertain to rhythm or meter.

The white cells are the untransposed pitch-classes of each voice in the order they are written/sounded. The colored cells below designate the prime hexachord of that row and present the operations at which it is combinatorial. I have separated each 12-tone row with a space in order to make it easier to notice patterns across their hexachords. I have chosen to leave out P_0 and R_6 for the First-Order hexachords (012345), (023457), and (024579) because they are always combinatorial at these transpositions. For (014589), the Third-Order hexachord, I have not included P_0, P_4, P_8, R_2, R_6, and R_T for the same reason. Omitting these transpositions from the table again makes it easier to notice patterns. Several of the lower cells

in Table 1.3 are not color-coded. The hexachords they represent do not have any combinatorial properties and are not frequent or significant enough to warrant their own colors.

As the Theme and Variation 1 sections contain half as much pitch content as the other four sections, I have presented them in a combined column in Table 1.1 (in addition to their separate columns). This column demonstrates how they produce complete 12-tone rows and coherent patterns when examined as a single unit at Level 1; Level 2 and 3 analyses do not require this additional column.

1.7 Level 1 Pattern A: Consistent Pitch Content of Prime Hexachords: Refer to Table 1.1. 12-tone rows of the same prime all-combinatorial hexachord, when divided down the middle (between the 6th and 7th note), produce the same two sets of pitch-classes. Those built from (012345) produce [6789TE] and [012345]; those from (023457), [E12346] and [5789T0]; those from (014589), [E03478] and [12569T]; and those from (024579), [68TE13] and [024579]. Exception: The Tenor and Bass voices in Variation 5 break this pattern, as they are built from (012345), yet produce [345678] and [9TE012]. This can be interpreted as the variation or modulation of the primary theme that closes the piece.

1.7 Level 1 Pattern B: ST and AB Exact Retrograde: When all SATB voices are built from the same prime, all-combinatorial hexachords in one section, (012345) in Theme/Variation 1 and (023457) in Variation 3, the first and second sets of six pitch-class sets exhibit several combinatorial relationships that maintain pitch order in addition to content. Hexachords in the Soprano voice have their exact retrogrades in the Tenor Voice; the Alto and Bass share the same relationship. Exception: There are two inconsistencies within this pattern. The second hexachords of the Alto and Bass Voices in Theme/Variation 1, and the second hexachords of the Soprano and Tenor in Variation 3, both contain errors in transposition. Although only two adjacent pitch-classes are reversed, these hexachords are no longer exact retrogrades. I doubt

these inconsistencies are errors in Babbitt's writing, but I cannot suggest any incentive Babbitt may have had to include them.

1.7 Level 1 Pattern C: Exact Hexachord Swap: When the prime hexachord of the Upper Voices is different from the Lower Voices, the first and second hexachords of the 12-tone row are swapped. The first hexachord of the Soprano is identical to the second hexachord of the Alto, and vice versa. The same relationship exists between the Tenor and Bass.

1.7 Level 1 Pattern D: Hexachord Interval Operations: When all SATB voices are built from the same prime, all-combinatorial hexachord in one section, the order of intervals within the four Upper Voice hexachords are combinatorial transpositions of each other. Designating the intervals from the Soprano's first hexachord as the primary interval order, the Soprano's second hexachord is the retrograde inversion, the Alto's first hexachord is the Retrograde, and the Alto's second hexachord is the inversion. These relationships also exist between the Tenor and Bass voices when all voices in a section are constructed of one all-combinatorial hexachord.

1.8 Level 2 Pattern A: Consistent Pitch Content Within Sections: Refer to Table 1.2. Similar to 1.7 Level 1 Pattern A,12-tone rows in one section, when divided in half, produce the same two sets of pitch-classes. Unlike this pattern at Level 1, the pitch content of first and second hexachords is consistent within each section. Although multiple sections share 12-tone row construction from one parent hexachord, the untransposed hexachord pitch content changes with every new section.

1.9 Level 3 Pattern A: Inconsistency Placement within Sections: Refer to Table 1.3. This pattern, although a not totally conclusive, is the only aspect of Level 3 that seems to have any role aside from supporting the rows of Levels 1 and 2. If the Theme and Variation 1 sections are ignored, each of the four remaining sections contains one hexachord that is unlike the rest. In Variation 2, the first 12-tone row is not constructed from (023457); in Variation 3, the

second 12-tone row is the only all-combinatorial hexachord based row; in Variation 4, the third 12-tone row does not contain any combinatorial properties; and in Variation 5, the fourth 12-tone row is not constructed from (024579). The order of these variances is 1-2-3-4, within their respective sections. There are no other obvious patterns at Level 3. Perhaps this is just a coincidence.

1.10 Levels 1 and 2 Pattern A: Reverse Hexachord Order: Refer to Tables 1.1 and 1.2. This pattern, similar to 1.9 Pattern A, is not as convincing as the prior few, but still seems worth mentioning. When the order of all-combinatorial hexachords between Levels 1 and 2 are compared, they are expressed in roughly the opposite order. Exception: Level 1's (024579) is paired with the semi-combinatorial (014568) in Level 2. Again, perhaps this is an unintentional coincidence, but this is hard to believe as Babbitt so meticulously adheres to 12-tone rules at the three levels of pitch analysis.

Section 2: Rhythmic and Metric Analysis:

2.1 Introduction to Rhythm and Meter: This section will primarily focus on the rhythmic content of each section, but will also briefly address the metric shift in Variation 3. In his program notes, Babbitt states that "all 16 possible partitions of the quarter unit in terms of the 16th note unit" are expressed within the first section. By reducing 16th-note rhythms to what he refers to as "partitions," Babbitt is able to apply a serial approach to the rhythmic aspect of this piece.

2.2 Definition of Partition: Outside of a musical context, the word "partition," is defined as a division, separation, or segregation. Babbitt states that there are 16 distinct partitions of the quarter note in terms of 16th-notes. When the quarter note is examined as a unit of musical time, it is equivalent in length/duration to four 16th-notes. The quarter note contains four

specific points in time at which a new 16th-note can be sounded; mathematically speaking, these are at the beginning of the beat, after 25% of the beat, after 50%, and after 75%. For analysis in *Semi-Simple Variations*, these four points within the quarter note can be interpreted as either containing a new note, or not containing a new note. If the four points/16th notes of any quarter note can only be one of those two things, there are 16 distinct possible combinations.

2.3 How to Interpret Appendix 2: The first table is a legend that pairs all 16 partitions with a Base 16 number or character. As 12-tone pitch content analysis generally uses the characters "T" and "E" to designate the tenth and eleventh pitch-classes, I have used "W," "X," "Y," and "Z" to designate the thirteenth, fourteenth, fifteenth, and sixteenth distinct rhythmic partitions. These partitions are arranged in a sparser-to-denser order. Class 0 represents a quarter-note that has no new notes sounded. Classes 1 through 4 contain one new 16th-note, Classes 5 through T contain two new 16th-notes, Classes E through Y contain 3, and Class Z contains new notes at all four 16th-note points within the quarter note. Appendix 2 does not contain any pitch content information.

Table 2.1 demonstrates the order of 16th-note partitions per quarter-note beat within each section. Columns are grouped by Babbitt's 3/4-2/4-3/4 measure scheme (except in Variation 3, in which measures are twice as long).

Table 2.2 reduces sets of eight partitions into their prime orders by the same method pitch-class sets are reduced. As the partitions present in Variation 3 are only 0, 5, T and Z, I have listed them by the amount of times they are used instead of in prime order, which is (016E) for the entire section. Table 2.2 presents data that depends on my order/grouping of the 16 distinct partitions. Unlike pitch-class order in this music, there is no convention for 16th-note rhythmic partition order; consequently, I was required to create my own. If these partitions are interpreted and ordered differently, the data in Table 2.2 would likely be different.

2.4 Rules for Interpreting Rhythmic Partitions: Like pitch content analysis of this piece, rhythmic partition analysis requires guidelines to address different aspects of the piece that create confusion. As repeated notes present challenges for pitch analysis, held notes do so for rhythmic study.

Rule 1: The determining rhythmic factor of all notes is the point at which they are played/sounded (Note-on in MIDI terms). The note's duration and point of release are of no importance.

Rule 2: Voice and pitch content are of no importance; a single pitch-class that is repeated multiple times is recorded as a new 16th-note division every time.

2.5 Pattern A: All 16th-note Partitions Present in Th, V1, and V2: Refer to Table 2.1. As stated in the program notes, all 16 distinct partitions are expressed in the Theme section. Although not in the same order, all partitions are again expressed in Variations 1 and 2. Despite the irregular meter scheme, 3/4-2/4-3/4-3/4-2/4-3/4, there are sixteen quarter-notes per section, each of which contains a unique partition. In the first three sections, no partitions are repeated until all other classes are expressed, in the same way pitch-class orders are written in this style of music.

2.5 Pattern B: Partition Rows Begin and End with O or Z: Refer to Table 2.1. The first and last rhythmic partitions are consistently Classes Z and 0. If the first partition of a row is Z, the last partition is 0, and vice versa. Classes 0 and Z are the extremes in my partition system; Class 0 does not contain any new notes within the quarter-note's four points, while Class Z contains new notes at all four points within the quarter note. The Theme, Variation 2, and Variation 4 all begin with Z and end with 0. Odd numbered Variations begin with 0 and end with Z; even though Variation 3 is metrically twice as long as every other section, it still follows this trend.

2.5 Pattern C: Partition-Class Selection of Variation 3: Refer to Table 2.1. Variation 3, metrically and rhythmically, is significantly different than the other sections. Aside from its variance in length, it is unique because it only contains four distinct partition-classes, which are 0, 5, T, and Z. As 0 and Z are extreme partitions, 5 and T are more central in my class system. These partitions both contain two new notes; 5's two notes are in the first half of the quarter-note, and T's are in the second half.

2.5 Pattern D: Partition-Class Distribution of Variation 3: Refer to Table 2.2. Similar to the pattern I have presented at 1.9, this pattern is not as conclusive as the others. I have chosen to call sets of eight partition-classes "Octachords" because it is consistent with the analysis of the piece's content that I have conducted. Clearly, groups of partition-classes do not produce chords, but "Octarhythms" and "Octagrooves" do not really make sense either.

Unlike the two prime octachords that can be reduced from the 16-partition rows of the other sections (like two prime hexachords in 12-tone rows), the octachords of Variation 3 only contain four distinct partition-classes and are identical at a prime level. The pattern in Variation 3 exists within the amount of times partition-classes are expressed.

If 0, 5, T, and Z are examined from an even more abstract perspective, where the amount of 16th-notes they each contain does not matter, they are simply four unlike pieces. Consider the amount: eight. If composed of a combination of four unlike pieces, there are only five ways this combination can be distributed across its parts. Equal distribution would have two of each unlike part, 2-2-2-2. The four other possible distributions that contain at least one of all four unlike parts are expressed within the four octachords of Variation 3. The first octachord's distribution scheme is 4-2-1-1, the second's is 3-2-2-1, the third's is 5-1-1-1, and the fourth's is 3-3-1-1. As stated above, this pattern is far-reaching.

2.5 Pattern E: No Single 16th-Note Partitions in V4 or V5: Refer to Table 2.1. Throughout the final two sections, rhythmic partition-classes that only contain one new 16th-

note within their corresponding quarter-note, 1-4 by number, are not expressed. Partition 6 is also absent from these sections.

Section 3: *Semi-Simple Variations* as Sonata Form:

3.1 Introduction to Form: This section will argue that *Semi-Simple Variations*, although packaged as a Theme and Variations by Milton Babbitt, is actually written in Sonata form that does not contain any repeats. All arguments in this section are based on data interpretation from Tables 1.1 and 2.1, which I suggest examining side by side while reading this analysis. Instead of describing patterns, this section will pair the qualities of a Sonata form with the correlating sections and material from *Semi-Simple Variations*.

3.2 Brief Summary of Terms: Sonata form consists of two distinct themes and key areas (generally reduced to A and B) that are expressed in these subsections: exposition, transition, development, and recapitulation. Unlike Rounded Binary form, which can also be reduced to AB, Sonata form contains clear transitions between A and B, as well as an unstable developmental section within B, before A is recapitulated. As Variations 2 and 4 play these roles, I have argued that the *Semi-Simple Variations* resembles a Sonata form more than a Rounded Binary. Finally, Theme and Variations form, as the name implies, presents one theme and several Variations that are different from, yet derived from, the Theme.

3.3 Theme/Variation1 and Variation 3- A and B Expositions: In this piece, (012345) is the primary theme, A, and (023457) is the contrasting theme, B. Both of these First-Order, all-combinatorial hexachords have their own sections at Level 1; the Theme and Variation 1 are completely built from (012345) as Variation 3 is from (023457). Although these sections both exhibit 1.7 Patterns A, B, and D regarding pitch content, they contain drastically different rhythmic partition-class schemes. Theme and Variation 1 both express all 16 rhythmic

partitionO-classes, and Variation 3 expresses only four as described in Section 2.5 Pattern D. Though rhythmically different, these expository sections are both metrically 32 beats/quarter-notes long. Theme and Variation 1, (012345), span 32 beats over the 3/4-2/4-3/4 metric scheme, while Variation 3, (023457), contains a metric shift that doubles measure length, 3/2-2/2-3/2, without changing tempo. As the Theme and Variation 1 and Variation 3 are similar in length and pitch content patterns, yet different in terms of rhythm and pitch content, they can be loosely compared to the I and V of a more traditional Sonata form piece.

3.4 Variation 2- The Transition: As Variation 2 simultaneously expresses 12-tone rows constructed of both (012345) and (02345) at Level 1, it is the transitional section between A and B. Traditionally, transitional material in a Sonata form occurs at the end of the A section; this explains its shared rhythmic scheme with Theme and Variation 1.

3.5 Variation 4- The Development: In Variation 4, two new all-combinatorial hexachords are introduced at Level 1; (014589) in the upper voices and (024579) in the lower voices. If we continue to compare (023457)/Variation 3 to V (the dominant), Variation 4 resembles a developmental section. In terms of pitch content, (024579) and (014589) get farther away from (012345) as the amount of chromaticism decreases. The word “unstable” is often used to describe harmonic movement in traditional developmental sections and is appropriate for (014589), the only all-combinatorial hexachord in this piece that is not First-Order. As (014589) is a Third-Order hexachord, it has three tritone axes and is thus less stable; unlike First-Order, all-combinatorial hexachords, (014589) produces identical pitch-class sets at two additional transpositional levels per combinatorial operation, including P_4 and P_8.

In terms of rhythm content, the partition-class scheme is similar to that of the Theme and Variations 1 and 2 as it expresses ten of the sixteen classes. As developmental sections often increase the momentum of the piece, the partition-classes expressed contain at least two new 16th-notes per quarter-note, which creates a livelier texture than any earlier sections.

3.6 Variation 5- Recapitulation: To close the piece, Babbitt briefly expresses (012345) in the lower voices. Although quite short, it is a clear return to a pitch-content related "home", or A', as it is often notated. However, the untransposed pitch is different than the (012345) constructed sets in the Theme and Variations 1 and 2. For the expository portion of (012345), the pitch-class sets are [012345] and [6789TE]; in the recapitulation, they are [345678] and [9TE012]. If the first sets are considered P_0, the second sets are P_3 or P_9. Although the recapitulated transpositions are only three semi-tones away from the expository sets, they are the most different untransposed pitch-class sets that can be constructed from the (012345) hexachord. (P_1 shares 5 pitch-classes, P_2 shares 4, P_3, shares 3, P_4 shares 4, P_5 shares 5, and P_6 shares 6.) To recall this paper's earlier analogy, this transposition can be examined as a variation or ornamentation of I, or the Tonic.

Section 4: Conclusion:

4.1 Thesis Argument: *Semi-Simple Variations*, a Theme and Variation piece by Babbitt's definition and presentation, exhibits more traits of the Sonata form. The piece's program notes and rehearsal letters package this piece as a Theme and five Variations, but its pitch, rhythmic, and metric content express distinct A and B material, instead of A, A', A'', A''', and so on. The only musical aspect of *Semi-Simple Variations* that consistently adheres to the Theme and Variations form is the number of measures per section, which is 6. (Again, the duration of Variation 3 is twice as long with its unusual metric shift.)

Although the pitch and rhythmic aspects of this piece clearly contain high levels of serialism, the difference in organization of the Theme/Variation 1 and Variation 3 is clearly not serial. If the piece was in fact a Theme and Variations, Variations 1 through 5 would likely

express their own unlike prime hexachords, untransposed pitch-class sets, partition-class schemes, perhaps other metric shifts or schemes, or a combination of all of these.

As this analysis has focused intensely on reducing the deep internal organization of many serialized aspects to data, it is also useful to examine it from a surface perspective and consider Babbitt's packaging of *Semi-Simple Variations'* score. First, it is necessary to acknowledge that Babbitt intentionally included the rehearsal markings and individual Variation titles in the score. This piece, despite its odd metric scheme, is only 36 measures long (lasting about 2 minutes) and is written for a solo performer. Both of these aspects make Babbitt's use of rehearsal markings unusual, if not excessive. It is also important to examine the titles of Babbitt's section/rehearsals: Theme, Variation 1, Variation 2, Variation 3, Variation 4, and Variation 5. If the Theme is considered 0, the sections spell out 0, 1, 2, 3, 4, 5: the primary theme. As Milton Babbitt was known to have infiltrated his highly intricate composition-style with a jovial personality, perhaps his presentation of this piece was simply an analytical joke or riddle.

4.2 Inconclusive/Unsolved Aspects of *Semi-Simple Variations:* As this piece is so meticulously constructed at so many levels, it is possible that I have misinterpreted some aspects of it, or simply failed to recognize them altogether. This analysis did not cover dynamics, texture, tempo, or register, all of which may contain patterns and/or serial derivation.

In addition to Appendices 1 and 2, I have included two other Appendices; Appendix 3 pairs the untransposed tetrachords of each 12-tone row at each pitch analysis level with their respective prime, and Appendix 4 does the same for all trichords. Both of these Appendices contain tables that are similar to those of Appendix 1, but without the combinatorial transpositions. These Appendices were not mentioned in the analysis, as they did not contribute to the central argument. They are included to aide anyone who chooses to study this piece in the future.

5.1 Further Questions: Over the course of my analysis, research, and data extrapolation, I have noticed several interesting composition choices Babbitt made within this piece. Some involve minute inconsistencies within data, and others are structural aspects I do not understand. I will list these questions below as starting points for further study.

Question 1: Why is the metric scheme 3/4-2/4-3/4-3/4-2/4-3/4 instead of four measures of 4/4? Both contain the same amount of beats/quarter-notes.

Question 2: What other aspects of this piece are constructed by serial methods? Note duration? Dynamics? Texture?

Question 3: Why were the other two all-combinatorial hexachords, (012678) and (02468T), not included anywhere in this piece? Why were the only all-combinatorial hexachords in this piece First- and Third-Order?

Question 4: Why is there only one instance of all four voices sounding new notes simultaneously? (Variation 4, Measure 1, Beat 3)

Question 5: Please refer to 1.7 Level 1 Pattern B. Why do two these similar inconsistencies exist?

Appendix 1: 12-Tone Rows, Prime Hexachords, and Their Combinatorial Transpositions

All-Combinatorial Hexachords	**012345***	**023457***	**024579***	**014589****
Semi-Combinatorial Hexachords	014568	014579	012346	

*(012345), (023457), and (024579) are 1st-Order All-combinatorial Hexachords, they are always Combinatorial at R6
**(014589), the only Third-Order All-Combinatorial Hexachord, is always Combinatorial at P4, P8, R2, R6, and RT

Table 1.1- Level 1: Four-Voice Part-Writing Style: Soprano, Alto, Tenor, Bass

	Theme	**Variation 1**	**Theme/Var. 1***	**Variation 2**	**Variation 3**	**Variation 4**	**Variation 5**
Soprano	T6E879	312504	T6E879 312504	T96E87 125034	E23614 T70895	8073E4 T59162	78E034 T96521
	I_9	**I_E**	**I_9, RI_3**	**I_9, RI_3**	**I_7, RI_1**	**$I_{3,7,E}$ $RI_{1,5,9}$**	**$I_{1,5,9}$ $RI_{3,7,E}$**
Alto	243051	7E69T8	243051 7E69T8	125034 T96E87	7T5890 632E41	T59162 8073E4	T96521 78E034
	I_1	**I_3**	**I_1, RI_7**	**I_3, RI_9**	**I_3, RI_9**	**$I_{3,7,E}$ $RI_{1,5,9}$**	**$I_{1,5,9}$ $RI_{3,7,E}$**
Tenor	978E6T	405213	978E6T 405213	3E4162 807T59	41632E 58907T	1E63T8 249057	534786 021T9E
	I_E	**I_9**	**I_E, RI_5**	**I_E, RI_5**	**I_9, RI_3**	**I_1, RI_7**	**I_1, RI_7**
Bass	150342	8T967E	150342 8T967E	807T59 3E4162	0985T7 14E236	249057 1E63T8	021T9E 534786
	I_3	**I_1**	**I_3, RI_9**	**I_1, RI_7**	**I_5, RI_E**	**I_5, RI_E**	**I_E, RI_5**

*the *Theme and V1* Column is not a real portion of *Semi-Simple Variations.* It demonstrates how the pitch-classes from each SATB voice in the *Theme* and *Variation 1* sections create complete 12-tone rows with complimentary, combinatorial hexachords; the notes they represent are not in the score

Table 1.2- Level 2: Division by Clef/Upper and Lower Voices/ Left and Right Hand: SA vs. TB

	Theme	**Variation 1**	**Variation 2**		**Variation 3**	
Upper Voices	T264E3 805791	37E612 590T48	1T9265 E08374	T12956 0E3874	E7T236 514890	6T372E 048915
	RI_3	RI_5	**$I_{1,5,9}$ $RI_{3,7,E}$**	**$I_{3,7,E}$ $RI_{1,5,9}$**	**$I_{3,7,E}$ $RI_{1,5,9}$**	**$I_{1,5,9}$ $RI_{3,7,E}$**
Lower Voices	915708 E364T2	408T95 6213E7	83E074 T16259	830E74 1T5692	049815 632TE7	514890 E7T236
	RI_5	RI_3	**$I_{3,7,E}$ $RI_{1,5,9}$**	**$I_{3,7,E}$ $RI_{1,5,9}$**	**$I_{1,5,9}$ $RI_{3,7,E}$**	**$I_{3,7,E}$ $RI_{1,5,9}$**
	Variation 4		**Variation 5**			
Upper Voices	T80759 13E642	T58907 13E642	T798E6 035214	7T8E96 520134		
	I_9, RI_3	**I_9, RI_3**	**I_9, RI_3**	**I_3, RI_9**		
Lower Voices	21E643 90T578	21E634 9T0578	502314 7T89E6	053214 T7896E		
	I_1, RI_7	**I_1, RI_7**	**I_7, RI_1**	**I_5, RI_E**		

Table 1.3- Level 3: All Notes, No Division by Voice/Part

Theme		**Variation 1**					
T29164 57E308	8E0356 749T12	3470E8 6T1952	569210 3TE478				
014579	014679/013689	**$I_{3,7,E}$ $RI_{1,5,9}$**	012569/013478				
Variation 2				**Variation 3**			
81T392 E65074	E0T183 672549	T8130E 295764	0E1T38 576492	04E798 T23165	516348 92TE07	516T34 78290E	04E78T 291365
012567	**I_1, RI_7**	**I_3, RI_9**	**I_E, RI_5**	013458	**I_E, RI_5**	012358/012457	012458
Variation 4				**Variation 5**			
21T8E0 756493	1930ET 654728	21T589 0E7634	9T130E 652478	T50927 3148E6	07T385 29E416	07T853 E29614	5T7829 10634E
RI_1	RI_3	012589/013478	RI_E	**I_E, RI_5**	**I_3, RI_9**	**I_3, RI_9**	012358/012457

*Hexachords in lower white cells do not exhibit any combinatorial properties
**When 12-tone rows are composed of two unlike hexachords, both primes are listed

Appendix 2: Rhythmic Partitions

Distinct 16th-Note Partitions of the Quarter Note

Rhythm																
Base 16 Classes	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**T**	**E**	**W**	**X**	**Y**	**Z**

Table 2.1: 16th-Note Partitions per Quarter Note

	3/4			2/4		3/4			3/4			2/4		3/4		
Theme	**Z**	**5**	**4**	**X**	**3**	**T**	**9**	**Y**	**7**	**1**	**E**	**6**	**2**	**8**	**W**	**0**
Variation 1	**0**	**X**	**8**	**3**	**9**	**Y**	**4**	**7**	**E**	**6**	**5**	**2**	**W**	**1**	**T**	**Z**
Variation 2	**Z**	**2**	**7**	**W**	**6**	**1**	**E**	**8**	**4**	**9**	**T**	**X**	**3**	**Y**	**5**	**0**
	3/2						2/2				3/2					
Variation 3.1	**0**	**0**	**0**	**Z**	**Z**	**5**	**T**	**0**	**Z**	**T**	**T**	**0**	**5**	**5**	**5**	**0**
Variation 3.2	**T**	**5**	**T**	**Z**	**0**	**T**	**T**	**T**	**5**	**Z**	**5**	**T**	**0**	**5**	**Z**	**Z**
	3/4			2/4		3/4			3/4			2/4		3/4		
Variation 4	**Z**	**5**	**7**	**Z**	**5**	**X**	**Z**	**Y**	**7**	**W**	**Z**	**T**	**8**	**Y**	**W**	**0**
Variation 5	**0**	**X**	**E**	**8**	**W**	**Z**	**T**	**7**	**E**	**Z**	**W**	**9**	**Z**	**7**	**T**	**Z**

Table 2.2: Octachords of Each Section and Their Primes

Theme	Z54X 3T9Y	71E6 28W0
	012678EW	**012678EW**
Variation 1	0X83 9Y47	E652 W1TZ
	012569EW	**012578EW**
Variation 2	Z27W 61E8	49TX 3Y50
	012569EW	**012578EW**
Variation 3	000Z Z5T0	ZTT0 5550
	0000 ZZ 5 T	**555 00 TT Z**
Variation 3.2	T5TZ 0TTT	5Z5T 05ZZ
	TTTTT 0 5 Z	**555 ZZZ 0 T**
Variation4	Z57Z 5XZY	7WZT 8YW0
	0128T	**012468**
Variation 5	0XE8 WZT7	EZW9 Z7TZ
	01345689	**023458**

Appendix 3: Tetrachords and Their Primes

Quadchords	0123	0124	0125	0126	0134	0135	0136	0137	0145	0146	0147	0148	0156	0157	0158	0167
Quadchords	0235	0236	0237	0247	0248	0257	0268	0347	0348	0358	0369	0459				

Table 3.1- Four-Voice Part-Writing Style: Soprano, Alto, Tenor, Bass

	Th/V1			V2			V3			V4			V5		
Soprano	T6E8	7931	2504	T96E	8712	5034	E236	14T7	0895	8073	E4T5	9162	78E0	34T9	6521
	0135	0268	0135	0125	0167	0125	0347	0369	0347	0459	0167	0459	0145	0167	0145
Alto	2430	517E	69T8	1250	34T9	6E87	7T58	9063	2E41	T591	6280	73E4	T965	2178	E034
	0124	0268	0124	0125	0167	0125	0235	0369	0235	0148	0268	0148	0145	0167	0145
Tenor	978E	6T40	5213	3E41	6280	7T59	4163	2E58	907T	1E63	T824	9057	5347	8602	1T9E
	0124	0268	0124	0135	0268	0135	0235	0369	0235	0247	0268	0247	0124	0268	0124
Bass	1503	428T	967E	807T	593E	4162	0985	T714	E236	2490	571E	63T8	021T	9E53	4786
	0135	0268	0135	0135	0268	0135	0347	0369	0347	0247	0268	0247	0124	0268	0124

Table 3.2- Division by Clef/Upper and Lower Voices/ Left and Right Hand: SA vs. TB

	Th.			V1			V2			V2.2			V3			V3.3		
Upper Voices	T264	E380	5791	37E6	1259	0T48	1T92	65E0	8374	T129	560E	3874	E7T2	3651	4890	6T37	2E04	8915
	0248	0347	0248	0348	0148	0248	0145	0167	0145	0145	0167	0145	0347	0135	0148	0347	0135	0148
Lower Voices	9157	08E3	64T2	408T	9562	13E7	83E0	74T1	6259	830E	741T	5692	0498	1563	2TE7	5148	90E7	T236
	0248	0347	0248	0248	0347	0248	0347	0369	0347	0347	0369	0347	0148	0135	0347	0347	0135	0348
	V4			V4.2			V5			V5.2								
Upper Voices	T807	5913	E642	T589	0713	E642	T798	E603	5214	7T8E	9652	0134						
	0135	0248	0247	0125	0137	0247	0123	0147	0134	0134	0347	0134						
Lower Voices	21E6	4390	T578	21E6	349T	0578	5023	147T	89E6	0532	14T7	896E						
	0237	0147	0235	0237	0167	0237	0235	0369	0235	0235	0369	0235						

Table 3.3- All Notes, No Division by Voice/Part

Th.			Th.2			V1			V1.2			V2			V2.2			V2.3			V2.4		
T291	6457	E308	8E03	5674	9T12	3470	E86T	1952	5692	103T	E478	81T3	92E6	5074	E0T1	8367	2549	T813	0E29	5764	0E1T	3857	6492
0145	0123	0347	0347	0123	0145	0347	0135	0148	0347	0235	0347	0257	0358	0237	0123	0125	0237	0257	0235	0123	0123	0135	0247
V3			V3.2			V3.3			V3.4			V4			V4.2			V4.3			V4.4		
04E7	98T2	3165	5163	4892	TE07	516T	3478	290E	04E7	8T29	1365	21T8	E075	6493	1930	ET65	4728	21T5	890E	7634	9T13	0E65	2478
0148	0126	0135	0135	0157	0125	0158	0145	0235	0158	0126	0135	0146	0157	0136	0236	0156	0146	0347	0134	0134	0146	0156	0146
V5			V5.2			V5.3			V5.4														
T509	2731	48E6	07T3	8529	E416	07T8	53E2	9614	5T78	2910	634E												
0237	0126	0247	0358	0147	0257	0135	0236	0358	0235	0125	0237												

Appendix 4: Trichords and Their Primes

Trichords	012	013	014	015	016	024	025	026	027	036	037	048

Table 4.1- Four-Voice Part-Writing Style: Soprano, Alto, Tenor, Bass

	Th.		V1		V2				V3				V4				V5			
Soprano	T6E	879	312	504	T96	E87	125	034	E23	614	T70	895	807	3E4	T59	162	78E	034	T96	521
	015	**012**	**012**	**015**	**014**	**014**	**014**	**014**	**014**	**025**	**025**	**014**	**015**	**015**	**015**	**015**	**014**	**014**	**014**	**014**
Alto	243	051	7E6	9T8	125	034	T96	E87	7T5	890	632	E41	T59	162	807	3E4	T96	521	78E	034
	012	**015**	**015**	**012**	**014**	**014**	**014**	**014**	**025**	**014**	**014**	**025**	**015**	**015**	**015**	**015**	**014**	**014**	**014**	**014**
Tenor	978	E6T	405	213	3E4	162	807	T59	416	32E	589	07T	1E6	3T8	249	057	534	786	021	T9E
	012	**015**	**015**	**012**	**015**	**015**	**015**	**015**	**025**	**014**	**014**	**025**	**027**	**027**	**027**	**027**	**012**	**012**	**012**	**012**
Bass	150	342	8T9	67E	807	T59	3E4	162	098	5T7	14E	236	249	057	1E6	3T8	021	T9E	534	786
	015	**012**	**012**	**015**	**015**	**015**	**015**	**015**	**014**	**025**	**025**	**014**	**027**	**027**	**027**	**027**	**012**	**012**	**012**	**012**

Table 4.2- Division by Clef/Upper and Lower Voices/ Left and Right Hand: SA vs. TB

	Th.				V1				V2				V2.2				V3				V3.2			
Upper Voices	T26	4E3	805	791	37E	612	590	T48	1T9	265	E08	374	T12	956	0E3	874	E7T	236	514	890	6T3	72E	048	915
	048	**015**	**037**	**026**	**048**	**015**	**037**	**026**	**014**	**014**	**014**	**014**	**014**	**014**	**014**	**014**	**014**	**014**	**014**	**014**	**037**	**037**	**048**	**048**
Lower Voices	915	708	E36	4T2	408	T95	621	3E7	83E	074	T16	259	830	E74	1T5	692	049	815	632	TE7	514	890	E7T	236
	048	**015**	**037**	**026**	**048**	**015**	**015**	**048**	**037**	**037**	**037**	**037**	**037**	**037**	**037**	**037**	**037**	**037**	**014**	**014**	**014**	**014**	**014**	**014**

	V4				V4.2				V5				V5.2			
Upper Voices	T80	759	13E	642	T58	907	13E	642	T79	8E6	035	214	7T8	E96	520	134
	024	**024**	**024**	**024**	**025**	**025**	**024**	**024**	**013**	**025**	**025**	**013**	**013**	**025**	**025**	**013**
Lower Voices	21E	643	90T	578	21E	634	9T0	578	502	314	7T8	9E6	053	214	T78	96E
	013	**013**	**013**	**013**	**013**	**013**	**013**	**013**	**025**	**013**	**013**	**025**	**025**	**013**	**013**	**025**

Table 4.3- All Notes, No Division by Voice/Part

Th.				Th.2				V1				V1.2			
T29	164	57E	308	8E0	356	749	T12	347	0E8	6T1	952	569	210	3TE	478
015	**025**	**026**	**037**	**014**	**013**	**025**	**014**	**014**	**014**	**037**	**037**	**014**	**012**	**015**	**014**
V2				**V2.2**				**V2.3**				**V2.4**			
81T	392	E65	074	E0T	183	672	549	T81	30E	295	764	0E1	T38	576	492
025	**016**	**016**	**037**	**012**	**027**	**015**	**015**	**025**	**014**	**037**	**013**	**012**	**027**	**012**	**027**
V3				**V3.2**				**V3.3**				**V3.4**			
04E	798	T23	165	516	348	92T	E07	516	T34	782	90E	04E	78T	291	365
015	**012**	**015**	**015**	**015**	**015**	**015**	**015**	**015**	**016**	**016**	**013**	**015**	**013**	**015**	**013**
V4				**V4.2**				**V4.3**				**V4.4**			
21T	8E0	756	493	193	0ET	654	728	21T	589	0E7	634	9T1	30E	652	478
014	**014**	**012**	**016**	**026**	**012**	**012**	**016**	**014**	**014**	**015**	**013**	**014**	**014**	**014**	**014**
V5				**V5.2**				**V5.3**				**V5.4**			
T50	927	314	8E6	07T	385	29E	416	07T	853	E29	614	5T7	829	106	34E
027	**027**	**012**	**025**	**025**	**025**	**025**	**025**	**025**	**025**	**025**	**025**	**025**	**016**	**016**	**015**